Unit-14 Millikan's Oil Droplet Experiment

Objective：

We will experimentally determine the unit charge which is a milestone in physics. (1 unit charge = 1.602×10-19 C)

Apparatus：

Millikan experiment apparatus(microscope, oil chamber, parallel plates, illumination system, oil atomizer, oil storage container, tripod), CCD camera, power supply(220 V)

Principle：

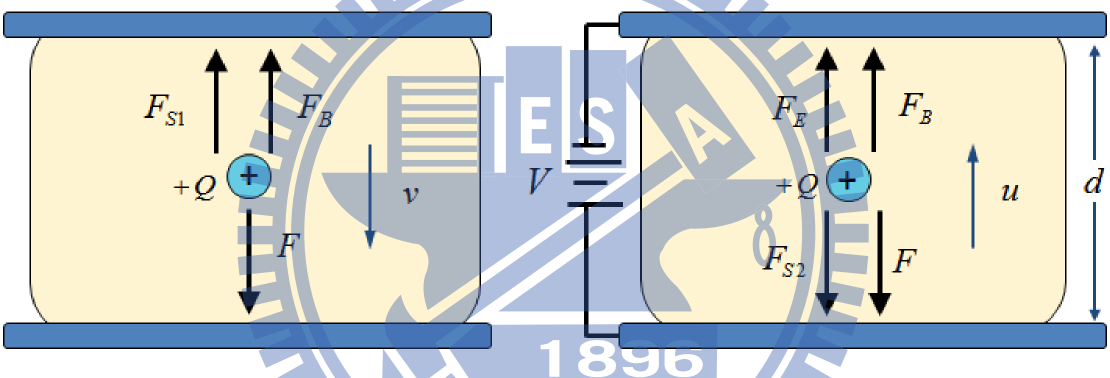

Figure 1. The force acting on a droplet (a) $E = 0$ (b) $E \neq 0$

In figure 1(a), droplet with charge Q, mass m_{oil}. The force acting on an oil can be described as follows.

(r is the radius of the droplet, ρ_{oil} is the density of oil, ρ_{air} is the density of air)

1. Gravity force： $F = m_{oil} g$, that $m_{oil} = \frac{4}{3}\pi r^3 \rho_{oil}$

2. Buoyant force： $F_B = m_{air} g$

   $m_{air} = \frac{4}{3}\pi r^3 \rho_{air}$, m_{air} is m_{oil} substituted with air density ρ_{air} .

3. Viscous force： $F_{S_1} = 6\pi r \eta v$

The viscous force on a spherical falling body, where η is the coefficient of viscosity and v is the falling velocity.

These forces acting on the oil droplets simultaneously can be described by Newton's laws of motion. The viscous force is balanced by gravity force and buoyant force acting on the droplet when the droplet reaches a terminal velocity. The equation of motion is

$$
\begin{aligned}
&F - F_B - F_{S1} = 0 \\
&\Rightarrow (m_{oil} - m_{air})g - 6\pi r\eta v = 0
\end{aligned}
\tag{1}
$$

Considering buoyant force and Gravity force, it is convenient to define the effective mass $m = m_{oil} - m_{air}$ and the effective density $\rho = \rho_{oil} - \rho_{air}$. Then, substitute them into equation (1). The terminal velocity of the falling drop is

$$
\begin{aligned}
&v = \frac{(m_{oil} - m_{air})g}{6\pi r\eta} \\
&\Rightarrow v = \frac{mg}{6\pi r\eta}
\end{aligned}
\tag{2}
$$

With the effective mass of oil droplet $\left(\frac{4\pi r^3}{3}\rho\right)$, we can get the radius of the droplet r is

$$
\begin{aligned}
&\left(\frac{4}{3}\pi r^3\rho\right)g - 6\pi r\eta v = 0 \\
&\Rightarrow r = \sqrt{\frac{9\eta v}{2\rho g}}
\end{aligned}
\tag{3}
$$

As shown in figure 1(b). The electric field is produced by two parallel plates maintained at a potential difference *V* and separated by a distance *d*. Thus, the electric field *E* is given by $E = \frac{V}{d}$.

When added with electric field, the force acting on an oil droplet with charge is electric force $F_E = QE$. (*E* is the electric potential across the two plates.)

Under the influence of electric field, the equation of motion is

$$
\begin{aligned}
&F + F_{S2} - F_B - F_E = 0 \\
&\Rightarrow (m_{oil} - m_{air})g - 6\pi r\eta u - QE = 0 \\
&\Rightarrow mg - 6\pi r\eta u - QE = 0
\end{aligned}
$$

With added electric field. The terminal velocity u of oil droplet is

$$u = \frac{mg - QE}{6\pi r\eta} \tag{4}$$

[Note] That the direction of u and v is defined as positive when the droplet is falling. Q may be positive or negative.

Following equation derived by substituting Equation (2) and (4), the charges Q on the droplet is

$$Q = -\frac{6\pi\, r\eta(u - v)}{E} = -\frac{6\pi\, r\eta d(u - v)}{V} \tag{5}$$

Substitute Equation (3) and (5). Then we can have the charges Q by the following question.

$$Q = -\frac{6\pi d\eta}{V}\sqrt{\frac{9\eta v}{2\rho g}}(u - v) \tag{6}$$

The parameters are shown as following (SI)：

$\eta = 1.81 \times 10^{-5}\ \mathrm{N/m^2}$ $\rho_{oil} = 875.3\ \mathrm{kg/m^3}$ $\rho_{air} = 1.29\ \mathrm{kg/m^3}$ $\rho = 874.01\ \mathrm{kg/m^3}$

Substitute these parameters into equation (6) and simplify it as following：

$$Q = -3.32 \times 10^{-8}\frac{\sqrt{v}(u - v)d}{V} \tag{7}$$

Therefore, by measuring the velocity of the oil droplet under different conditions, the charge carried by an oil droplet can be determined. Assume Q on the oil droplet is composed of several charges, that is

$Q = ne$ (e represents elementary charge)

As shown in figure 2. We make a plot of the number of times a value of Q occurred vs. Q. The quantization of electric charges can be recognized from the fact that these charges of all oil droplets are grouping around integer multiples of elementary charge e.

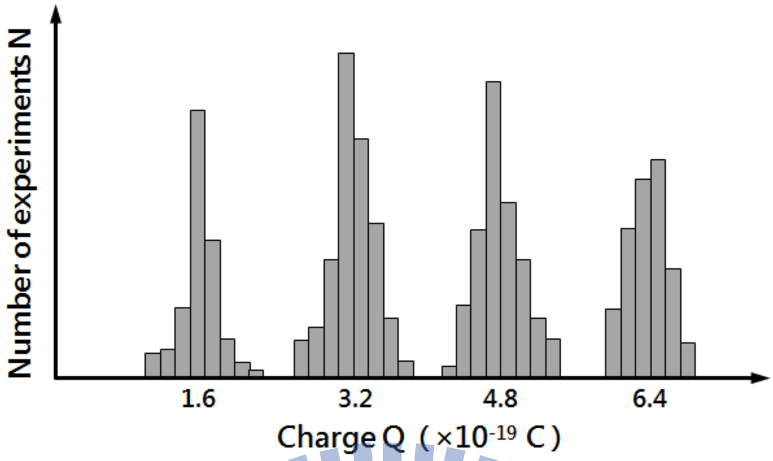

Figure 2. Number of experiments N – Charge Q diagram

Remarks：

1. Do not take off the atomizer. Request the teaching assistant to add some oil if you need.
2. Before turning on any power, make sure you have wired things correctly.

Procedure：

- **Preparation**
  1. Set up the Millikan apparatus as shown in figure 3.
  2. Plug in the electricity and give the capacitors positive and negative voltage, turn the power on and provide the illumination.
  3. Press the atomizer to spray oil droplets into the chamber. Focus the microscope and get a clear image of oil droplets and scales.
  4. Set up CCD camera and adjust the horizontal position. Then, connect CCD to computer with transmission line.
  5. Focus the microscope eyepiece direct to CCD camera lens.
  6. Start the Cyberlink PowerDirector Software. Press capturing and save the video on the desktop.
  7. Focus the CCD camera and display the image of the droplets and the scale on the monitor as clearly as possible. Make sure that the scale on the microscope is vertical.

Figure 3. Experiment set-up

A. Recording and Transferring the video

1. Set the voltage to 600 V and then switch off the voltage directly.
2. Press the atomizer to spray oil droplets into the chamber. Wait few seconds, then start to record the image
3. As the oil droplets move to the middle view of the image, turn the voltage on and record the time. Wait the oil droplets reach the terminal velocity and keep recoding few seconds, then stop recording.

B. Analyzing terminal velocity of oil droplets through Logger Pro Software

1. Open the software and set the parameters by instruction book.
2. Choose an oil droplet and record terminal velocity with electric field or not.
3. The terminal velocity was be enlarged, because of the oil droplet was be enlarged by objective lens and the eye lens.
4. But the oil droplet and the ruler were be enlarged 10X simultaneously, so we just consider that was be enlarged 1.875X by objective lens. As shown in Figure 4.

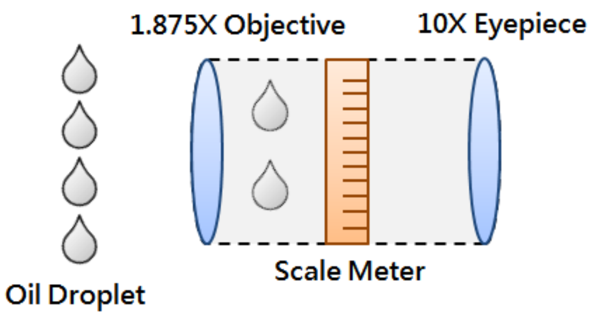

Figure 4. The relative position of oil chamber and microscope

5. Calculating the velocity v of oil drop without electric field and the velocity u of oil drop under electric field.

$$v = \frac{v'}{1.875} \qquad u = \frac{u'}{1.875}$$

C. Calculating the charge Q of oil dropt

1. Take v and u into equation to calculate charge Q.
2. Choose 10 different oil droplet and repeat above steps.
   [Note] If you cannot find the suitable oil droplet to observe, please record it again.
3. Repeat the above steps with setting voltage to 400 V.

[Reference]

1. The relation between total charge Q and elementary charge e is $Q = ne(n = 1,2,3...)$. Choose the minimum total charge Q from your data and set its n=1.
2. Dividing other data by minimum Q and record the number of charge n.
3. As shown in figure 5. Plot the total charge Q versus to the number of charge n.

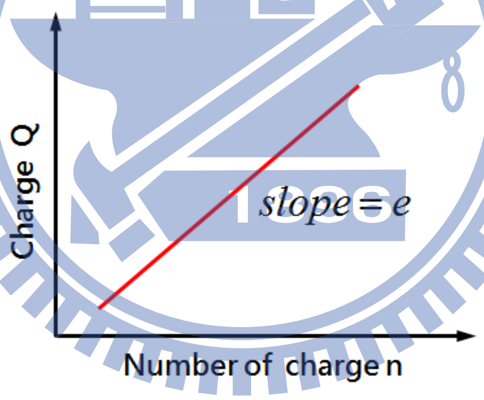

Figure 5. Charge Q-Number of charge n diagram

<u>Questions</u>：

1. How to determine that the oil is positively or negatively charged? Please explain.
2. If this experiment operate in higher or less than 1-atm. Can we get the same result? Please explain.
3. In this experiment, if the water droplets to replace the oil droplets, whether the basic charge can still be obtained? Please explain.