Unit-11　Ratio of Specific Heats of a Gas

Objective：

Using Ruchhardt's Method to measure the ratio of specific heat of gas.

Apparatus：

Atmosphere, low-pressure sensor, analogy device, rubber tube

Principle：

A.　Absorption and liberation of heat at system

In this experiment, system is the gas, when the environmental temperature greater then system temperature ($T_E > T_S$), the system absorbed heat from outside. At this time, we define the heat dQ is positive that the system absorbed; on the other hand, when $T_E < T_S$, the heat flow into surrounding from the system, it means exothermal, now dQ is negative. As shown in figure 1.
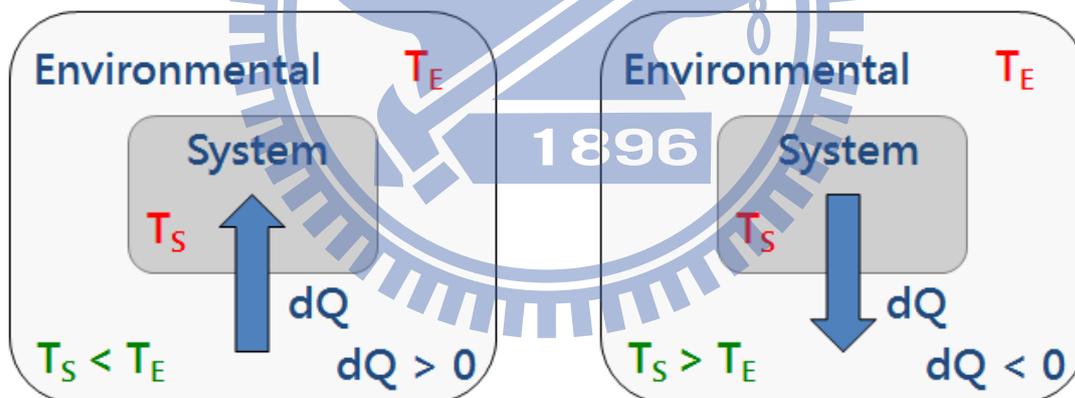

Figure 1. Absorption and liberation of heat at system

B.　The first Law of thermodynamics　$dE_{int} = dQ - dW = dQ - PdV$

E_{int} is the internal energy at the system, Q is the heat, W is the work done on the surroundings by the system. For the ideal gas, its internal energy is just related to temperature, that is $dE_{int} = dE_{int}(T)$.

C. Adiabatic process for ideal gas

In "adiabatic process", work exerted on the air will be totally converted into the internal energy, and hence the pressure and temperature will rise; on the other hand, when the air expands, it does work on the surroundings. Therefore, the internal energy of air decreases and the pressure and temperature decrease.

D. Isothermal process for ideal gas

In "isothermal process", the internal energy at the system isn't change; we would know that $dQ = dW$ from the first Law of thermodynamics. When the air is compressed, the surroundings do work on the air. Therefore the pressure rises, and the system has to transfer energy out to the surroundings to maintain the same temperature ($dQ < 0$). On contrary, when the system (air) expands, it does work on the surroundings. The pressure decreases, and hence the system has to absorb heat ($dQ > 0$) to maintain the temperature.
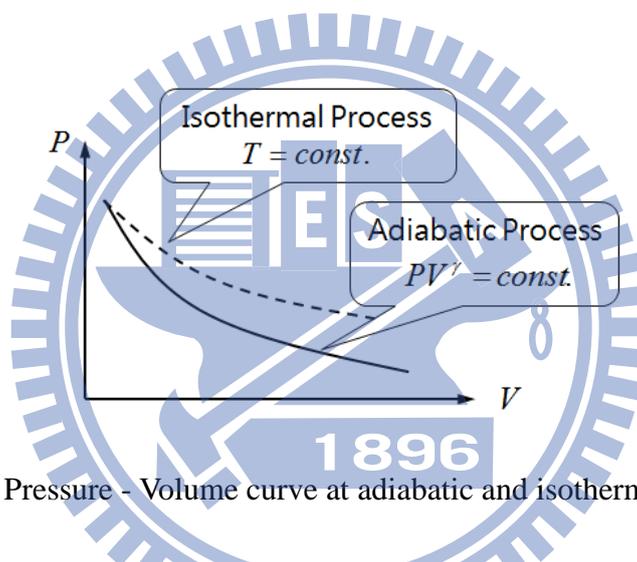

Figure 2. Pressure - Volume curve at adiabatic and isothermal process

From 1st law of thermodynamics, $dE_{int} = dQ - dW = dQ - PdV$, P changes more rapidly in adiabatic process than in isothermal process. i.e. $P\text{-}V$ curve is more steep in adiabatic process (as shown in the following figure).

Consider an ideal gas, from 1st law of thermodynamics, we know the heat dQ flow into the gas, the difference of internal energy dE_{int}; and the work done on the surroundings by the system is PdV. It would be written:

$$dQ = dE_{int} + PdV \tag{1}$$

E. Molar specific heat

If the volume of the gas remains a constant, then the heat to rise one degree per mole gas is called " molar specific heat at constant volume"— C_v. From equation (1), we have

$$C_V = \left(\frac{dQ}{dT}\right)_V = \left(\frac{\partial E_{int}}{\partial T}\right)_V = \frac{dE_{int}}{dT}$$

$$\Rightarrow C_V dT = dE_{int} \qquad (2)$$

By equation (1) and (2) could get

$$dQ = C_V dT + PdV \qquad (3)$$

From ideal gas equation $PV = nRT$. When $n = 1$ we can get

$$PdV + VdP = RdT \qquad (4)$$

By equation (3) and (4) could get

$$dQ = C_V dT + RdT - VdP$$
$$= (C_V + R)dT - VdP \qquad (5)$$

If the pressure of the gas remains a constant, then the heat to rise one degree per mole gas is called " molar specific heat at constant pressure"— C_p. Into equation (5), that $dP = 0$

$$dQ = (C_V + R)dT$$

$$C_P = \left(\frac{dQ}{dT}\right)_P = C_V + R \qquad (6)$$

F. Ratio of specific Heats γ

The ratio of molar specific heat is defined as the ratio of molar specific heat at constant pressure - C_P to molar specific heat at constant volume - C_V, that is

$$\gamma \equiv \frac{C_P}{C_V} = \frac{C_V + R}{C_V}$$

It can also be calculated by statistical mechanics. In statistical mechanics, each degree of freedom contributes, on average, $\frac{1}{2}RT$ of energy per molecule. In our life, most gases are diatomic gases, which the molecules have the shape of a dumbbell. In this model, there are 5 degrees of freedom; three are related to the translational motion and two related to rotational motion; the vibrational motion along the molecule axis is neglected in this case. Therefore, one mole of air have $\frac{5}{2}RT$ of internal energy, i.e. $C_V = \frac{5}{2}R$, and $C_P = C_V + R = \frac{7}{2}R$.

Hence the theoretical value γ is 1.40.

$$\gamma \equiv \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{5}{2}R + R}{\frac{5}{2}R} = \frac{7}{5} = 1.40$$

G. Adiabatic gas equation

In thermodynamics, an adiabatic process is one that occurs without transfer of heat or matter between a thermodynamic system and its surroundings. In an adiabatic process, energy is transferred to its surroundings only as work.

A process that does not involve the transfer of heat or matter into or out of a system, so that Q = 0, is called an adiabatic process, and such a system is said to be adiabatically isolated.

$$dE_{int} = -dW = -PdV \tag{7}$$

Substitute equation (7) into equation (2)

$$C_V dT + PdV = 0 \tag{8}$$

By ideal gas formula ($n = 1$)

$$PV = RT \Rightarrow P = \frac{RT}{V} \tag{9}$$

Substitute equation (9) into equation (8)

$$C_V dT + \frac{RT}{V} dV = 0$$

$$\Rightarrow \frac{dT}{T} + \frac{R}{C_V} \frac{dV}{V} = 0 \tag{10}$$

From equation (10), we can get

$$ln\,T + ln\,V^{R/C_V} = const.$$
$$\Rightarrow TV^{R/C_V} = const. \qquad (11)$$

By ideal gas formula ($n = 1$)

$$PV = RT \Rightarrow T = \frac{P}{R}V \qquad (12)$$

Substitute equation (12) into equation (11)

$$\frac{P}{R}V \cdot V^{R/C_V} = const.$$
$$\Rightarrow PV \cdot V^{R/C_V} = PV^{R+C_V/C_V} = const.$$

then we can get

$$PV^{C_P/C_V} = const. \qquad (13)$$
$$\Rightarrow PV^{\gamma} = const.$$

As show in figure 3, press the piston with appropriate power quickly, the piston will move downward x and receives the upward pressure. The piston will upward movement because of received the upward pressure, gas pressure shake with time.

This process without heat enters in or flows out, so the process is adiabatic process. And the gas in the apparatus of law-atmosphere is satisfied adiabatic process gas equation $PV^{\gamma} = const..$
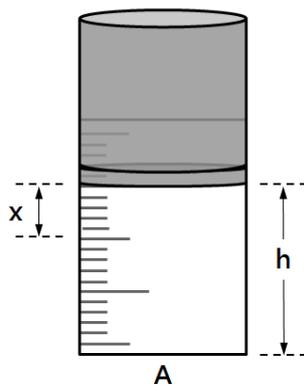

Figure 3. Experimental illustration

The Ruchhardt experimental technique may obtain the air ratio of specific heats is, the inferential reasoning process is as follows. Do the differential operation on the equation (13).

$$V^\gamma dP + \gamma P V^{\gamma-1} dV = 0$$
$$\Rightarrow dP = -\frac{\gamma P}{V} dV$$

(14)

Substitute $dV = xA$ into equation (14), we can get

$$dP = -\frac{\gamma P}{V} xA = -\frac{\gamma P A x}{V}$$

The net force on piston

$$F = (dP)A$$
$$= \left(-\frac{\gamma P A x}{V}\right)A = -\left(\frac{\gamma P A^2}{V}\right)x$$

The net force and movement of piston x satisfy simple harmonic case $F = -kx$, so

$$k = \frac{\gamma P A^2}{V}$$

The fluctuation period of the pressure of piston is (And m is the mass of piston.)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mV}{\gamma P A^2}}$$

(15)

H. Ratio of Specific Heats of a Gas γ

In the experiment, the piston cross-sectional area is A, the mass of piston is m, and P_0 is an atmosphere pressure. (Show in figure 4)

(a) Promotes the piston after suitable height h, close the valve that is not connects with the pressure sensor.

(b) The total gas pressure P is sum of an atmosphere pressure and the pressure of piston.

$$P = P_0 + \frac{mg}{A}$$

Figure 4. Experiment set-up

(c) The total air volume V is sum of the air volume of piston(underneath scale zero) Ah, the air volume in switch valve rubber pipe (underneath piston) and connection low pressure sensing rubber pipe air volume v'.

$$V = Ah + v' \qquad (16)$$

(d) From equation (15) and (16), the relation between the height h and the shake period of piston is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mV}{\gamma PA^2}}$$

$$\Rightarrow V = \frac{\gamma A^2 P T^2}{4\pi^2 m} = Ah + v' \qquad (17)$$

$$\Rightarrow h = \left(\frac{\gamma A P}{4\pi^2 m}\right)T^2 - \frac{v'}{A}$$

(e) Plot the diagram of $h - T^2$, from the $slope = \dfrac{\gamma A P}{4\pi^2 m}$, we can get the ratio of molar specific heat.

$$\gamma = \frac{4\pi^2 m(slope)}{AP}$$

Remarks：

1. In the experiment you need to seize the piston by your hand after raising the piston to certain altitude. Close the valve disconnected with the low-pressure sensor by using another hand.

2. Beware of the piston if it drops after you close the valve. The reason of piston dropping is because the valve disconnected with the low-pressure sensor does not completely closed.

Procedure：

1. Open the software and set the related parameters by instruction book.
2. Open all valves and raise the piston to appropriate altitude $h = 90.0$ mm and then size the piston by one hand. Close the valve connected with the low-pressure sensor by using another hand. Loosen the piston and record the altitude of piston.
3. Press start button to start acquiring data, meanwhile press the piston with appropriate power quickly.
4. When the 『Pressure-Time』 diagram don't change, press start button to stop acquiring data.
5. Record five periods time, then calculate the period T.
6. Change altitude of piston and restart steps.
7. Plot the diagram of $h - T^2$, and find the slope by linear regression analysis.
8. Calculate the ratio of molar specific heat of air.
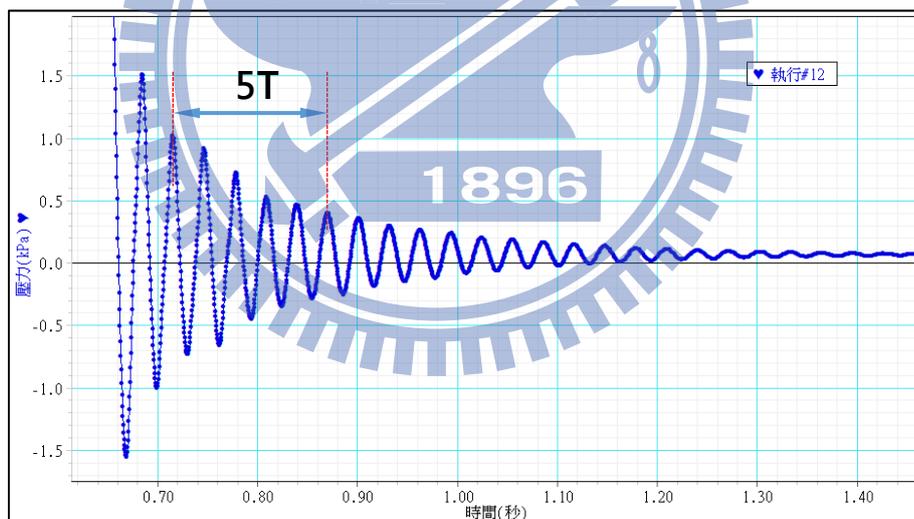
Questions：

1. Would the slope of the $h - T^2$ graph for Helium be greater or less than the slope for air? Please explain.
2. What is the ingredient of air? Why this experiment assume air as double atoms molecule. Please explain.