Mean and Error Transfer：

Two or more experimental results of the arithmetic, should be consider the error transfer.

Let $x = \bar{x} \pm \sigma_x$ 且 $y = \bar{y} \pm \sigma_y$

A. Error Transfer of Addition and Subtraction

$$\overline{x \pm y} = \bar{x} \pm \bar{y} \text{ ， } \sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2$$

$$x \pm y = (\bar{x} \pm \bar{y}) \pm \sigma_{x+y} \text{ ， } \sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$$

In general form：

$$\sigma_N{}^2 = \sum_{i=1}^{n} \sigma_i^2$$

B. Error Transfer of Addition and Subtraction of Multiplication and Division

$$\overline{xy} = \bar{x} \times \bar{y} \text{ ； } \sigma_{xy} = \sqrt{\left[\left(\frac{\sigma_x}{\bar{x}}\right)^2 + \left(\frac{\sigma_y}{\bar{y}}\right)^2\right]} \times (\bar{x} \times \bar{y})$$

$$x \times y = (\bar{x} \times \bar{y}) \pm \sigma_{xy}$$

$$\overline{\frac{x}{y}} = \frac{\bar{x}}{\bar{y}} \text{ ； } \sigma_{\frac{x}{y}} = \sqrt{\left[\left(\frac{\sigma_x}{\bar{x}}\right)^2 + \left(\frac{\sigma_y}{\bar{y}}\right)^2\right]} \times \frac{\bar{x}}{\bar{y}}$$

$$\frac{x}{y} = \left(\frac{\bar{x}}{\bar{y}}\right) \pm \sigma_{\frac{x}{y}}$$

In general form：

$$\left(\frac{\sigma_N}{y}\right)^2 = \left(\frac{\sigma_1}{y_1}\right)^2 + \left(\frac{\sigma_2}{y_2}\right)^2 + \cdots + \left(\frac{\sigma_n}{y_n}\right)^2$$

Where y is the average derived and y_1, y_2, … y_n are average of every element in the calculation.

C. The Relation of Rtandard Deviation before and after Calculation

$$\overline{x^l \times y^m} = (\bar{x}^l \times \bar{y}^m) \; ; \; \left(\frac{\sigma_{x^l y^m}}{\bar{x}^l \times \bar{y}^m}\right)^2 = l^2\left(\frac{\sigma_x}{\bar{x}}\right)^2 + m^2\left(\frac{\sigma_y}{\bar{y}}\right)^2$$

$$x^l \times y^m = (\bar{x}^l \times \bar{y}^m) \pm \sigma_{x^l y^m}$$

D. General Standard Deviation

Set $N = f(x,y)$, and then

$$\sigma_N = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2}$$

Least Square Regression Analysis：

It is an analysis instrument in common use. By fitting the optimized regression we are able to minimize the sum of square of vertical distance to the regression.

Be given the data set:

$$(x_i, y_i) \text{，} i = 1,2,3,\ldots n$$

A. Linear Regression

If the optimized regression is the form of $y = f(x) = Ax + B$ with A and B unknown, then the sum of square of vertical distance to the regression is:

$$D(A,B) = \sum_{i=1}^{n} \left[f(x_i) - y_i\right]^2 = \sum_{i=1}^{n} \left[Ax_i + B - y_i\right]^2$$

The so-called optimization is to decrease $D(A,B)$:

$$\frac{\partial D}{\partial A} = 0 \; ; \; \frac{\partial D}{\partial B} = 0$$

$$\frac{\partial D}{\partial A} = \sum_{i=1}^{n} \frac{\partial}{\partial A}\left[Ax_i + B - y_i\right]^2 = \sum_{i=1}^{n} 2\left[Ax_i + B - y_i\right]x_i$$

$$= 2\sum_{i=1}^{n}\left[Ax_i^2 + Bx_i - x_i y_i\right] = 2\left[A\sum_{i=1}^{n} x_i^2 + B\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i y_i\right] = 0$$

$$\frac{\partial D}{\partial B} = \sum_{i=1}^{n} \frac{\partial}{\partial B}\left[Ax_i + B - y_i\right]^2 = 2\left[\sum_{i=1}^{n} Ax_i + \sum_{i=1}^{n} B - \sum_{i=1}^{n} y_i\right]$$

$$= 2\left[A\sum_{i=1}^{n} x_i + nB - \sum_{i=1}^{n} y_i\right] = 0$$

And the solutions are

$$A = \frac{n\left[\sum_{i=1}^{n} x_i y_i\right] - \left[\sum_{i=1}^{n} x_i\right]\cdot\left[\sum_{i=1}^{n} y_i\right]}{n\left[\sum_{i=1}^{n} x_i^{\ 2}\right] - \left[\sum_{i=1}^{n} x_i\right]^2} \ ; \ B = \frac{\left[\sum_{i=1}^{n} y_i\right] - A\left[\sum_{i=1}^{n} x_i\right]}{n}$$

We can just input (x_i, y_i) to engineering calculator or software like MS-excel, and then A and B can be obtained.

B. Exponential Regression

It describes the data in form of $y = A \times e^{Bx}$. Also it can be applied through calculator or software.

C. Logarithmic Regression

It describes the data in form of $y = A \times \ln x + B$. Also it can be applied through calculator or software.

D. Power Regression

It describes the data in form of $y = A \times x^B$. Also it can be applied through calculator or software.